For $v \neq 0$, $(v+\varepsilon\dot{v})^{-1} = v^{-1} - \varepsilon\dot{v}/v^2$, as direct multiplication verifies. Addition, multiplication and this inverse therefore reproduce all three rules. The limit argument above proves that these formal coefficients coincide with actual derivatives.

Comparative cost and preferred method

For a straight-line calculation with L elementary operations, first-order jet evaluation performs a constant number of operations for each original operation, hence $O(L)$ arithmetic and the same dependency ordering. Repeated finite-difference calls do not prove any rule. For the requested theorem, the conventional limit proof is already efficient; the jet algebra becomes useful when the rules are reused in automatic differentiation.