Source task and conventions

Deduce the constant multiple rule as a special case of the product rule.

Conventional solution and proof

Take the constant function $g(x) = c$. Its derivative is zero by the defining difference quotient. Therefore $(cf)' = g'f + gf' = 0f + cf' = cf'$. This argument also covers $c = 0$.

Structural or analytic derivation

Multiplication of a first-order jet by c sends (f, f') to (cf, cf'). This is the linearity of scalar multiplication in the jet algebra, and it gives the same identity without additional differentiation rules.

Comparative cost and preferred method

Both proofs require one application of an already proved linear or product identity. The numerical derivative evaluation requires one multiplication by c. No asymptotic improvement is available in this fixed scalar task.