Source task and conventions

Differentiate $f(x) = x^x$ on its real differentiable domain $x > 0$.

Conventional solution and proof

Taking logarithms gives $\log f(x) = x \log x$. Differentiating yields $f'/f = \log x + 1$, and hence

$$\boxed{f'(x) = x^x(1 + \log x), \quad x > 0.}$$

The steps are valid because $f(x) > 0$ and log is differentiable on $(0, \infty)$.

Structural or analytic derivation

For a local complex logarithm L with $L(x) = \log x$, set $F(z) = \exp(zL(z))$. Since $L'(z) = 1/z$, the chain rule gives $F' = F(L + 1)$. Restricting to positive real z recovers the displayed result. This extension also supplies analytic derivatives of every order in a disk avoiding the logarithm cut.

Comparative cost and preferred method

Logarithmic differentiation evaluates the result with one logarithm, one exponential/power and elementary arithmetic. Expanding a difference quotient is more cumbersome but is not the strongest conventional baseline. The conventional logarithmic method and the analytic continuation have the same formula and arithmetic cost. The real logarithmic solution is preferable for this question.