Source task and conventions

Plot $f(x) = x^3 - x^{-1}$ and its tangent at $x = 1$.

Conventional solution and proof

The domain is $\mathbb{R} \setminus \{0\}$. Direct differentiation gives $f'(x) = 3x^2 + x^{-2}$. Since $f(1) = 0$ and $f'(1) = 4$, the tangent is

$$\boxed{y = 4(x - 1).}$$

The exact difference factors as

$$f(x) - 4(x - 1) = \frac{(x - 1)^2(x^2 + 2x - 1)}{x}.$$

It is $O((x - 1)^2)$ near one, proving tangency independently of the derivative calculation. The graph uses separate intervals on either side of zero; the vertical asymptote is not joined by a plotted line.