Structural or analytic derivation

The Laurent representation $z^3 - z^{-1}$ makes the excluded point and the derivative $3z^2 + z^{-2}$ explicit. A Taylor expansion about one has first-order coefficient four and the exact quadratic remainder displayed above. This analytic description certifies both the tangent and the singularity shown in the figure.

Comparative cost and preferred method

Both symbolic methods use a fixed number of operations. Drawing m graph samples costs $\Theta(m)$ under either representation. The Laurent formulation improves the singularity check, while direct differentiation is the simplest conventional solution.