Source task and conventions

Construct the dependency graph of $f(x) = \log(x^2)\sin x + x^{-1}$, for real $x \neq 0$.

Conventional solution and proof

Use the six intermediate quantities

$$v_1 = x^2, \quad v_2 = \log v_1, \quad v_3 = \sin x, \quad v_4 = v_2 v_3, \quad v_5 = x^{-1}, \quad v_6 = v_4 + v_5 = f.$$

The directed dependencies are $x \to v_1 \to v_2 \to v_4 \to v_6$, $x \to v_3 \to v_4$, and $x \to v_5 \to v_6$. Since $x \neq 0$, $v_1 > 0$ and all six operations are defined. The accompanying graph includes every edge and every intermediate quantity.

Structural or analytic derivation

This graph is a straight-line program. It also defines a first-order jet program by replacing each scalar by a value–tangent pair. The number and direction of dependencies are unchanged. Locally near any nonzero real point, $\log(z^2)$ has a consistent analytic branch; global branch choices cannot be omitted for complex inputs.

Comparative cost and preferred method

The shared expression uses six elementary nodes. Recomputing x^2, $\log(x^2)$ or $\sin x$ for each requested derivative can increase work; storing the node values avoids it. A competent conventional expression graph already performs this sharing, so the proposed graph has no separate asymptotic advantage. Its benefit is an explicit, checkable dependency contract.