Source task and conventions

Differentiate the graph for $f(x) = \log(x^2)\sin x + x^{-1}$ and label the local derivatives.

Conventional solution and proof

The edge derivatives are

$$\frac{\partial v_1}{\partial x} = 2x, \quad \frac{\partial v_2}{\partial v_1} = v_1^{-1}, \quad \frac{\partial v_3}{\partial x} = \cos x,$$

$$\frac{\partial v_4}{\partial v_2} = v_3, \quad \frac{\partial v_4}{\partial v_3} = v_2, \quad \frac{\partial v_5}{\partial x} = -x^{-2}, \quad \frac{\partial v_6}{\partial v_4} = \frac{\partial v_6}{\partial v_5} = 1.$$

Summing products along the three directed paths from x to v_6 gives

$$\boxed{f'(x) = \frac{2\sin x}{x} + \log(x^2)\cos x - \frac{1}{x^2}.}$$

Each path term is the chain rule; the sum is the derivative of the addition node. This proves the result on both real half-lines.

Structural or analytic derivation

The same derivative is the coefficient of ε in the six-node jet evaluation. The product node produces two contributions, and the reciprocal produces the negative term. This gives an algebraic check that no path contribution was omitted. The graph figure prints each of the seven local edge labels.

Comparative cost and preferred method