Source task and conventions

Prove $A^{\mathsf{T}} + B^{\mathsf{T}} = (A + B)^{\mathsf{T}}$ for equal-shaped matrices.

Conventional solution and proof

For every i, j, $[(A + B)^{\mathsf{T}}]_{ij} = A_{ji} + B_{ji} = [A^{\mathsf{T}} + B^{\mathsf{T}}]_{ij}$. Equality of all entries establishes the claim. Equal shapes are required for the sum to be defined.

Structural or analytic derivation

Coordinate permutation is linear: it commutes with coordinatewise addition. This proves the identity as a statement about linear maps on the space of matrices. It also justifies moving a transpose through an addition in a compiler's expression graph.

Comparative cost and preferred method

Either parenthesization performs mn scalar additions for a dense output. A view-based transpose adds only constant metadata; a copying transpose can add one or two mn-element copies. Thus the arithmetic count is tied, with potential memory-traffic savings determined by the implementation contract.