This is exactly the definition of symmetry. No assumption that A was symmetric is used.

Structural or analytic derivation

The map $P(A) = (A + A^{\mathsf{T}})/2$ is a projection onto the symmetric subspace: $P(P(A)) = P(A)$ follows from the proved symmetry. Moreover $A - P(A) = (A - A^{\mathsf{T}})/2$ is skew-symmetric. Their Frobenius inner product is zero because each off-diagonal pair cancels and the skew diagonal vanishes. Hence P is the orthogonal symmetrization projection.

Comparative cost and preferred method

The source proof uses two reusable identities. Forming the whole output costs $\Theta(n^2)$, which is optimal for a dense $n \times n$ output. Symmetric storage needs $n(n+1)/2$ entries instead of n^2, but only when the required output accepts that representation. The basic proof is already optimal in explanatory economy without a meaningful timing metric.