Source task and conventions

Solve the supplied probability exercise with prior 0.0015, sensitivity 0.99, false-positive probability 0.1, conditional independence when $H = 1$, and $P(D_1 = D_2 = 1 \mid H = 0) = 0.02$.

Conventional solution and proof

Under $H = 0$, both marginals have positive probability 0.1. Subtracting the common $(1, 1)$ cell gives

	$D_2 = 0$	$D_2 = 1$
$D_1 = 0$	0.82	0.08
$D_1 = 1$	0.08	0.02

.

Under $H = 1$, independence gives cells $0.0001, 0.0099, 0.0099, 0.9801$ in the same order. For one positive test,

$$P(H = 1 \mid D_1 = 1) = \frac{0.0015(0.99)}{0.0015(0.99) + 0.9985(0.1)} = \frac{297}{20267} \approx 0.014654.$$

For two positive tests,

$$\boxed{P(H = 1 \mid D_1 = D_2 = 1) = \frac{0.0015(0.9801)}{0.0015(0.9801) + 0.9985(0.02)} = \frac{29403}{428803} \approx 0.068570.}$$

Each denominator is the total probability of the conditioning event, obtained by partitioning on H. All four cells sum to one and have the required marginals.

Structural or analytic derivation