Conventional solution and proof

Linearity of expectation gives $\mathbb{E}(\alpha^{\mathsf{T}} s) = \alpha^{\mathsf{T}} \mu$. Centering and expanding the square give

$$\operatorname{Var}(\alpha^{\mathsf{T}} s) = \alpha^{\mathsf{T}} \Sigma \alpha.$$

With nonnegative weights, expected return is a convex combination of the means and is maximized by allocating to an index of maximal mean; ties allow any mixture of maximizers. The printed budget constraint alone does not impose nonnegative weights. If shorting is unrestricted and two means differ, weights $te_i + (1-t)e_j$ make expected return unbounded as t tends to the appropriate infinity. A precise risk-constrained formulation is

$$\max_{\alpha} \ \mu^{\mathsf{T}} \alpha \quad \text{subject to} \quad \mathbf{1}^{\mathsf{T}} \alpha = 1, \quad \alpha^{\mathsf{T}} \Sigma \alpha \le V,$$

with $\alpha \ge 0$ appended when a long-only portfolio is intended. The covariance is positive semidefinite because $v^{\mathsf{T}} \Sigma v = \mathbb{E}[v^{\mathsf{T}}(s-\mu)]^2 \ge 0$, so the variance constraint is convex.

Structural or analytic derivation

For the budget-only model with $\Sigma \succ 0$, define $a = \mathbf{1}^{\mathsf{T}} \Sigma^{-1} \mathbf{1}$, $b = \mathbf{1}^{\mathsf{T}} \Sigma^{-1} \mu$, $d = \mu^{\mathsf{T}} \Sigma^{-1} \mu - b^2/a$, and $\alpha_0 = \Sigma^{-1}\mathbf{1}/a$. Let $v = \Sigma^{-1}(\mu - b\mathbf{1}/a)$. Every budget-feasible $\alpha = \alpha_0 + w$ has $\mathbf{1}^{\mathsf{T}} w = 0$ and variance $1/a + w^{\mathsf{T}} \Sigma w$. Cauchy–Schwarz in the Σ inner product gives $\mu^{\mathsf{T}} w = v^{\mathsf{T}} \Sigma w \le \sqrt{d}\sqrt{w^{\mathsf{T}} \Sigma w}$. For $V \ge 1/a$ and $d > 0$, equality is attained uniquely by

$$\alpha^* = \alpha_0 + \sqrt{(V - 1/a)/d}\, v.$$

If $V < 1/a$ the feasible set is empty. If $d = 0$, all budget-feasible portfolios have the same expected return. This proves the closed-form optimum under the stated additional positive-definite, unrestricted-sign assumptions; it does not solve an arbitrary long-only active-set problem.