Geometric coefficient extraction yields the same closed form. The two poles also explain the dominant 5^n growth and alternating correction.

Comparative cost and preferred method

Enumerating all tilings takes at least a_n output operations. Counting the prefix by recurrence takes $n-1$ updates. A single remote coefficient uses one integer power by $O(\log n)$ multiplications, plus a parity decision and division by six. The characteristic-root baseline already obtains this formula; the generating function supplies an equivalent proof. For the first six terms, the recurrence is simpler than repeated powers.