Source task and conventions

Give the recursive count and solve it by characteristic roots for two square colours and three domino colours.

Conventional solution and proof

There is one empty tiling and two tilings of length one. Partitioning by the last tile proves $a_0 = 1$, $a_1 = 2$ and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 2$. The characteristic roots of $r^2 - 2r - 3$ are 3 and -1. The candidate $c3^n + d(-1)^n$ meets the initial conditions when $c = 3/4, d = 1/4$. Therefore

$$\boxed{a_n = (3^{n+1} + (-1)^n)/4.}$$

Both basis sequences satisfy the recurrence, and induction proves that this initial-value solution is unique.

Structural or analytic derivation

The generating function is $A(z) = (1 - 2z - 3z^2)^{-1} = \frac{3/4}{1-3z} + \frac{1/4}{1+z}$. Extracting the coefficient proves the same expression and records the two exponential modes.

Comparative cost and preferred method

The source explicitly requires characteristic roots, so that derivation is retained. It is as efficient symbolically as partial fractions. A prefix of length N uses $O(N)$ recurrence updates; a single large-index term uses $O(\log n)$ integer multiplications by exponentiation. No new advantage over the prescribed method is asserted.