Source task and conventions

Solve $a_n = 2a_{n-1} - a_{n-2}$, impose the stated initial values, and classify initial values giving $a_9 = 30$ or $a_9 = x$.

Conventional solution and proof

Rearrange to $a_n - a_{n-1} = a_{n-1} - a_{n-2}$. The first differences are constant, so telescoping gives

$$a_n = a_0 + n(a_1 - a_0).$$

This proves the complete two-parameter solution. With $(a_0, a_1) = (1, 2)$ it is $a_n = n + 1$. The condition $a_9 = 30$ is exactly $9a_1 - 8a_0 = 30$. Over the reals, choose any $a_0 = u$ and $a_1 = (30 + 8u)/9$. For integer initial values all solutions are $a_0 = 3 + 9t$, $a_1 = 6 + 8t$ with $t \in \mathbb{Z}$: the first equation modulo nine gives $a_0 \equiv 3$, and substitution gives the second. Every real x is attainable with real initial values by $a_0 = a_1 = x$; with integer initial values, precisely integer x are attainable. With nonnegative initial values, a nonnegative constant choice attains every nonnegative x in the same scalar domain.

Structural or analytic derivation

The generating function is

$$A(z) = \frac{a_0 + (a_1 - 2a_0)z}{2} = \frac{a_0}{} + (a_1 - a_0)\frac{z}{2}.$$