The double pole produces an affine polynomial in the index. Coefficient extraction gives the same formula, explaining the repeated-root case without a limiting argument between two equal roots.

Comparative cost and preferred method

The first-difference proof is the strongest conventional baseline and uses one invariant. Evaluating a_n needs one multiplication and one addition after the initial difference is formed, in unit-cost scalar arithmetic. The generating-function derivation adds setup and does not improve this evaluation. The preferred solution is telescoping.