Conventional solution and proof

From $G(z) = 1/(1-z) = \sum_{n\ge 0} z^n$, formal differentiation gives $G'(z) = 1/(1-z)^2 = \sum_{n\ge 0}(n+1)z^n$. Scaling, shifting and replacing z give the complete table:

(a)	$4/(1-z)$
(b)	$2/(1-z)^2$
(c)	$2z^3/(1-z)^2$
(d)	$1/(1-5z)$
(e)	$1/(1+3z)$
(f)	$1/(1-5z^2)$
(g)	$z/(1-z^3)^2$.

For (c), multiplication by z^3 inserts exactly three zeros. For (f), the coefficient at $2k$ is 5^k and odd coefficients vanish. For (g), the coefficient at $3k+1$ is $k+1$ and the other two residue classes vanish. These descriptions reproduce every term, not just the displayed prefixes.

Structural or analytic derivation

All series are ordinary generating functions $A(z) = \sum_{n\ge 0} a_n z^n$. Identities hold formally: the coefficient of any fixed power uses finitely many operations. When complex-analytic evaluation is desired, the same identities hold inside the nearest pole. The geometric identity follows by multiplying $\sum_{n\ge 0} z^n$ by $1-z$; coefficient cancellation leaves one.

Comparative cost and preferred method