so

$$\boxed{A(z) = \frac{4}{1-z} + \frac{z}{(1-z)^3}.}$$

Coefficient extraction gives $a_n = 4 + n(n+1)/2$, whose first differences and initial value verify the intended sequence at every index.

Structural or analytic derivation

A rational pole decomposition separates a constant component from the triangular component. The formula $A(z) = (4 - 7z + 4z^2)/(1-z)^3$ is equivalent and can be checked by bringing the preceding terms to a common denominator.

Comparative cost and preferred method

The fixed-order recurrence and output-size comparison in AC-035 applies to this coefficient sequence. The calculation above supplies its particular initial data and rational function.