Source task and conventions

Use multiplication to obtain the generating function of $S_n = \sum_{k=0}^{n} F_k$.

Conventional solution and proof

The Cauchy product with $1/(1-z)$ has coefficient $\sum_{k=0}^{n} F_k$, so

$$\boxed{S(z) = \frac{z}{(1-z)(1-z-z^2)}.}$$

The recurrence gives $F_k = F_{k+2} - F_{k+1}$. Telescoping from zero to n therefore proves $S_n = F_{n+2} - 1$, consistent with $S_0 = 0$. This also verifies the product coefficient independently.