Source task and conventions

Find the generating function for $a_n = 2 \cdot 5^n + 7(-3)^n$.

Conventional solution and proof

Summing the two geometric series separately gives

$$\boxed{A(z) = \frac{2}{1-5z} + \frac{7}{1+3z} = \frac{9-29z}{1-2z-15z^2}.}$$

The coefficient of z^n in the first expression is the specified closed formula. The rational identity follows by multiplying out its numerator. It also gives $a_0 = 9, a_1 = -11$ and $a_n = 2a_{n-1} + 15a_{n-2}$.

Structural or analytic derivation

All series are ordinary generating functions $A(z) = \sum_{n\geq 0} a_n z^n$. Identities hold formally: the coefficient of any fixed power uses finitely many operations. When complex-analytic evaluation is desired, the same identities hold inside the nearest pole. The geometric identity follows by multiplying $\sum_{n\geq 0} z^n$ by $1 - z$; coefficient cancellation leaves one.

Comparative cost and preferred method