Conventional solution and proof

The second summand contributes one at every index. The first contributes zero at index zero and $3 \cdot 4^{n-1}$ at index $n \geq 1$. Therefore

$$\boxed{a_0 = 1, \qquad a_n = 1 + 3 \cdot 4^{n-1} \quad (n \geq 1).}$$

This follows from $3z \sum_{k \geq 0} 4^k z^k = \sum_{n \geq 1} 3 \cdot 4^{n-1} z^n$. Extending the positive-index formula to zero would give an incorrect value.

Structural or analytic derivation

All series are ordinary generating functions $A(z) = \sum_{n \geq 0} a_n z^n$. Identities hold formally: the coefficient of any fixed power uses finitely many operations. When complex-analytic evaluation is desired, the same identities hold inside the nearest pole. The geometric identity follows by multiplying $\sum_{n \geq 0} z^n$ by $1 - z$; coefficient cancellation leaves one.

Comparative cost and preferred method

The fixed-order recurrence and output-size comparison in AC-035 applies to this coefficient sequence. The calculation above supplies its particular initial data and rational function.