Source task and conventions

Compute a_7 for $A(z) = 2(1-z)^{-2}z(1-z-z^2)^{-1}$.

Conventional solution and proof

The factors have coefficients $2(k+1)$ and F_j, respectively. Hence

$$a_7 = 2\sum_{j=0}^{7}(8-j)F_j = 2(0+7+6+10+12+15+16+13) = \boxed{158}.$$

The Cauchy product is finite at degree seven, so no convergence assumption is needed.

Structural or analytic derivation

Twice cumulative summation gives the general identity

$$a_n = 2\sum_{k=0}^{n}\sum_{j=0}^{k}F_j = 2\sum_{k=0}^{n}(F_{k+2}-1) = 2(F_{n+4}-n-3).$$

The telescoping Fibonacci sum proves the last equality. At $n=7$, $F_{11}=89$ gives $2(89-10)=158$.

Comparative cost and preferred method