Source task and conventions

Explain why $(1-z)^{-2}$ generates $1, 2, 3, 4, \ldots$.

Conventional solution and proof

Multiply two geometric series. The coefficient of z^n is the number of ordered pairs of nonnegative integers (i, j) with $i + j = n$. The pairs are $(0, n), (1, n-1), \ldots, (n, 0)$, exactly $n + 1$ of them. Therefore $(1-z)^{-2} = \sum_{n \geq 0} (n+1) z^n$.

Structural or analytic derivation

Alternatively, formally differentiating $1/(1-z) = \sum_{n \geq 0} z^n$ gives the same expansion after shifting the summation index. Formal differentiation is valid coefficientwise; on $|z| < 1$ the analytic power series can also be differentiated term by term.

Comparative cost and preferred method

The fixed-order recurrence and output-size comparison in AC-035 applies to this coefficient sequence. The calculation above supplies its particular initial data and rational function.