Source task and conventions

Use $(1 - z - z^2)^{-1} = 1 + z + 2z^2 + 3z^3 + \cdots$ to identify the four sequences.

Conventional solution and proof

Let $b_n = F_{n+1}$, with $F_0 = 0, F_1 = 1$. The requested functions give:

$$\begin{array}{l|l} (a) & 0, 0, 1, 1, 2, 3, 5, \ldots; \quad a_n = 0 \ (n < 2),\ a_n = F_{n-1} \ (n \geq 2) \\ (b) & 1, 0, 1, 0, 2, 0, 3, 0, 5, \ldots; \quad a_{2k} = F_{k+1},\ a_{2k+1} = 0 \\ (c) & 1, 3, 18, 81, 405, \ldots; \quad a_n = 3^n F_{n+1} \\ (d) & 1, 2, 4, 7, 12, 20, \ldots; \quad a_n = F_{n+3} - 1. \end{array}$$

Parts (a)–(c) follow from multiplication by z^2, substitution z^2, and substitution $3z$. In (d), multiplication by $(1 - z)^{-1}$ gives $\sum_{k=0}^{n} F_{k+1}$, which telescopes to $F_{n+3} - 1$. These arguments establish all coefficients.

Structural or analytic derivation

All series are ordinary generating functions $A(z) = \sum_{n \geq 0} a_n z^n$. Identities hold formally: the coefficient of any fixed power uses finitely many operations. When complex-analytic evaluation is desired, the same identities hold inside the nearest pole. The geometric identity follows by multiplying $\sum_{n \geq 0} z^n$ by $1 - z$; coefficient cancellation leaves one.

Comparative cost and preferred method