Multiplication by $1 + 2z$ cancels every positive-degree coefficient and leaves one, proving the formal identity.

Structural or analytic derivation

The pole at $z = -1/2$ records the alternating sign and exponential magnitude. Analytically the power series converges for $|z| < 1/2$; the formal coefficient identity does not require evaluating z.

Comparative cost and preferred method

The fixed-order recurrence and output-size comparison in AC-035 applies to this coefficient sequence. The calculation above supplies its particular initial data and rational function.