Source task and conventions

Find the generating function for $1, 1, 1, 2, 3, 4, 5, 6, \ldots$.

Conventional solution and proof

The first two coefficients are one. Beginning at index two, the coefficients are $n - 1$. Therefore

$$\boxed{A(z) = 1 + z + \frac{z^2}{(1-z)^2}.}$$

Indeed, $z^2 \sum_{k \geq 0} (k+1) z^k$ contributes $1, 2, 3, \ldots$ precisely at indices two onward. This verifies the offset and all tail coefficients.

Structural or analytic derivation

The formal-series convention and geometric-series justification are established in AC-036. The displayed transformation uses that same coefficientwise arithmetic on its stated convergence domain.

Comparative cost and preferred method