These identities follow from $\left(\begin{smallmatrix}1&1\\1&0\end{smallmatrix}\right)^k = \left(\begin{smallmatrix}F_{k+1}&F_k\\F_k&F_{k-1}\end{smallmatrix}\right)$, proved by induction, and squaring. Recursing on $\lfloor n/2 \rfloor$ returns (F_n, F_{n+1}) exactly.

Comparative cost and preferred method

The source's own generating-function approach is retained as the standard proof. Linear recurrence computation uses $\Theta(n)$ integer additions; fast doubling uses $O(\log n)$ multiplication-level stages. Since F_n has $\Theta(n)$ bits, these are not bit-complexity bounds or constant-time closed forms. The nearest-integer theorem requires enough precision to separate the true value from the half-integer boundary.