Structural or analytic derivation

The square-root singularity at $z = 1/4$ explains the exponential factor 4^n and the algebraic factor $n^{-3/2}$. Exact coefficients also satisfy

$$b_{n+1} = \frac{2(2n+1)}{n+2} b_n, \quad b_0 = 1,$$

by dividing adjacent factorial formulas. Each division is exact because the values equal the proved integer tree counts. This recurrence is the useful computational consequence of the algebraic generating function.

Comparative cost and preferred method

The convolution recurrence uses $N(N+1)/2$ products to produce $b_1, \ldots, b_N$. The multiplicative recurrence uses N updates, each with small-index multiplication and exact division, reducing coefficient-operation growth from quadratic to linear. The source's closed formula and a competent combinatorial implementation already support this reduction. The asymptotic estimate alone is not an exact-integer algorithm.