Source task and conventions

Convert a nonzero-node point-value representation of A into one for its degree-bound-n coefficient reversal.

Conventional solution and proof

From the definition,

$$A_{\mathrm{rev}}(x) = \sum_{j=0}^{n-1} a_{n-1-j} x^j = x^{n-1} A(1/x), \quad x \neq 0.$$

For every supplied pair (x_i, y_i) with $x_i \neq 0$, form

$$\boxed{\left(x_i^{-1},\ x_i^{-(n-1)} y_i\right).}$$

Indeed $A_{\mathrm{rev}}(1/x_i) = x_i^{-(n-1)} A(x_i)$. Distinct original nodes remain distinct after inversion. The transformed nodes need not be the original nodes, which is permitted in a point-value representation.

Structural or analytic derivation

This is pullback under the inversion map on the Riemann sphere together with the degree weight x^{n-1}. The weight makes the reversed polynomial regular at zero even though the formula uses $1/x$ away from zero. Its constant term is a_{n-1}, so the map's apparent singularity is removable for the polynomial.

Comparative cost and preferred method