Source task and conventions

Prove that fewer than n distinct values cannot uniquely specify a polynomial of degree below n.

Conventional solution and proof

Suppose $m < n$ distinct pairs (x_i, y_i) are consistent with one polynomial A of degree below n. Let $R(x) = \prod_{i=1}^{m}(x - x_i)$, with $R = 1$ for $m = 0$. For every scalar c, the polynomial $A_c = A + cR$ still has degree below n and satisfies all the pairs. Since R is nonzero, different c give different polynomials over an infinite field such as $\mathbb{R}$ or $\mathbb{C}$. Thus uniqueness fails. Conversely, if two degree-below-n polynomials agree at n distinct points, their difference has n roots and degree below n, hence is zero: repeated factorization by each distinct linear factor proves this elementary root bound.