Structural or analytic derivation

Evaluation at m nodes is a linear map from an n-dimensional coefficient space to an m-dimensional value space. Its kernel contains the nonzero polynomial R and in fact has dimension $n - m$, since every degree-below-n vanishing polynomial is divisible by R. This gives the exact ambiguity dimension.

Comparative cost and preferred method

The result is an information lower bound: at least n independent scalar values are required for arbitrary degree-below-n coefficients. Neither complex evaluation nor faster arithmetic removes it. The constructive vanishing-polynomial proof is the most direct conventional certificate.