Source task and conventions

Compute the DFT of $(0, 1, 2, 3)$ with the CLRS convention $\omega_4 = e^{2\pi i/4} = i$.

Conventional solution and proof

By $y_k = \sum_{j=0}^{3} a_j i^{jk}$,

$$y_0 = 6, \quad y_1 = i - 2 - 3i = -2 - 2i, \quad y_2 = -1 + 2 - 3 = -2, \quad y_3 = -i - 2 + 3i = -2 + 2i.$$

Thus $\boxed{y = (6, -2 - 2i, -2, -2 + 2i)}$. The inverse formula $a_j = \frac{1}{4} \sum_k y_k i^{-jk}$ recovers $(0, 1, 2, 3)$; cancellation of the nonconstant geometric sums proves that inverse formula.

Structural or analytic derivation

Split into even input $(0, 2)$ and odd input $(1, 3)$. Their two-point transforms are $(2, -2)$ and $(4, -2)$. Combining with twiddles $1, i$ yields the same four outputs. This is the complete radix-two butterfly calculation.

Comparative cost and preferred method

For four inputs, direct arithmetic is already modest, and the butterfly mainly exposes reusable symmetry. The general radix-two recurrence is $T(n) = 2T(n/2) + \Theta(n) = \Theta(n \log n)$ instead of $\Theta(n^2)$ direct sums. Many software libraries use the opposite exponential sign; comparing outputs requires this explicit convention.