Structural or analytic derivation

Equivalently, use the same recursive butterfly with conjugate roots and divide by n once at the top level. The normalized DFT is unitary because its rows are orthonormal by the same sum identity. Unitarity gives a norm-preserving exact transform; it does not eliminate floating-point rounding in its implementation.

Comparative cost and preferred method

One forward FFT plus two conjugation passes and one scaling pass costs $\Theta(n \log n)$. Normalization at every recursive level is unnecessary. This is the standard efficient inverse transform, so the PHI interpretation supplies a proof and conditioning information with no additional asymptotic speedup.