Conventional solution and proof

(a) Euclidean division by $x - z$ has the form $A = (x - z)Q + r$ with constant r. Substitution gives $r = A(z)$. (b) Hence $Q_{kk} = A(x_k)$. The polynomial $P_{0,n-1}$ has degree n, greater than $\deg A$, so its remainder is A itself. (c) Since P_{ik} divides P_{ij}, the difference $A - Q_{ij}$ is divisible by P_{ik}. Thus A and Q_{ij} have the same unique remainder modulo P_{ik}, proving $Q_{ik} = Q_{ij} \bmod P_{ik}$. The same argument applies to P_{kj}, including the shared endpoint in the source notation. (d) Build a balanced product tree for the factors $x - x_k$. At the root store A; at each node reduce its stored remainder modulo each child product. At a leaf the result is the constant $A(x_k)$ by (a). Induction down the tree, using (c), proves that each stored value equals the remainder of the original A modulo that node's product.

Structural or analytic derivation

The algorithm passes local residue classes down a divisibility hierarchy. In its implementation, child index ranges are disjoint, $[i, k]$ and $[k + 1, j]$; this is compatible with the source's more general divisibility identity. Repeated evaluation points do not invalidate the remainder argument, though repeated leaves return repeated values.

Comparative cost and preferred method

At level l, the total degree is $O(n)$ and the largest division size is $O(n/2^l)$. Under the source's remainder-cost assumption, that level costs $O(n \log(n/2^l))$. Summing the logarithmic levels yields $O(n \log^2 n)$; the product tree has the same bound. Separate Horner evaluations cost $\Theta(n^2)$. At roots of unity, the more specialized FFT costs only $O(n \log n)$ and remains preferable.