Structural or analytic derivation

The determinant is an alternating polynomial in the nodes and vanishes whenever two nodes coincide. The product of differences displays the complete zero divisor; the column proof above fixes the multiplicative constant and sign. Distinctness of the nodes is therefore exactly the invertibility condition for polynomial interpolation.

Comparative cost and preferred method

Evaluating the product uses $n(n-1)/2$ differences and one fewer product after initialization, with no $n \times n$ matrix formation. Generic elimination costs $\Theta(n^3)$ arithmetic and $\Theta(n^2)$ storage. This structured determinant formula proves an arithmetic reduction to $\Theta(n^2)$ for the requested scalar output. Near-colliding nodes can still cause numerical sensitivity.