with zero iterations when the initial error already meets the target. This is $O(\log\log(1/\delta))$ for fixed admissible C, e_0. If $M = 0$, the affine function is solved in one step. The condition $e_0 < 1$ alone does not imply $Ce_0 < 1$. (d) For $f(x) = (x-3)^2$, Newton gives $x_{t+1} - 3 = (x_t - 3)/2$ and $e_t = 2^{-(t+1)}$. Thus the exact tolerance count is $\max\{0, \lceil \log_2(1/(2\delta)) \rceil\}$. The root is multiple and violates the nonzero-derivative hypothesis at x_*. Gradient descent with step η gives $e_t = \frac{1}{2}|1-2\eta|^t$. It converges for $0 < \eta < 1$, has the same rate at $\eta = 1/4$, and reaches the minimizer in one step at $\eta = 1/2$. Without a specified step size there is no unique numerical comparison.

Structural or analytic derivation

The Newton map is a dynamical system. At a root of known multiplicity two, the modified map $x \mapsto x - 2f/f'$ reaches three in one step for this exact quadratic. The conventional gradient method with the exact quadratic step also reaches it in one step. A root certificate or multiplicity calculation can justify the modification, while a generic claim of quadratic convergence at a multiple root is false. The graph shows both linear Newton convergence and the one-step quadratic-model methods.