Source task and conventions

Solve CLRS Problem 2-3: runtime, a naive evaluator and the full loop-invariant proof of Horner's rule.

Conventional solution and proof

(a) The printed loop performs $n+1$ iterations with constant scalar work, so its runtime is $\Theta(n)$. (b) A direct-from-scratch evaluator is:

```
p = 0
for k in range(n+1):
    power = 1
    for j in range(k):
        power *= x
    p += A[k] * power
return p
```

It performs $\sum_{k=0}^{n} k = n(n+1)/2$ power multiplications plus $n+1$ coefficient multiplications, hence $\Theta(n^2)$ arithmetic. (c) At the start of the descending iteration i, assert $p = \sum_{k=0}^{n-i-1} A[k+i+1]x^k$, with an empty sum interpreted as zero. Initially $i = n$ and $p = 0$, so the invariant holds. The update makes

$$A[i] + xp = A[i] + \sum_{k=1}^{n-i} A[k+i]x^k = \sum_{k=0}^{n-i} A[k+i]x^k,$$

which is exactly the invariant for the next iteration $i-1$. On termination the next index would be -1, giving $p = \sum_{k=0}^{n} A[k]x^k = P(x)$. Thus initialization, preservation and termination are all established.