Source task and conventions

Prove weak duality for integer programs, give a counterexample to strong duality, and prove $IP \le P = D \le ID$.

Conventional solution and proof

Use the source's standard primal $\max c^{\mathsf{T}}x$ subject to $Ax \le b$, $x \ge 0$, and dual $\min b^{\mathsf{T}}y$ subject to $A^{\mathsf{T}}y \ge c$, $y \ge 0$. (a) For any feasible pair, including integer pairs,

$$c^{\mathsf{T}}x \le (A^{\mathsf{T}}y)^{\mathsf{T}}x = y^{\mathsf{T}}Ax \le y^{\mathsf{T}}b.$$