The first inequality uses $x \geq 0$, the second $y \geq 0$, proving weak duality without any continuity assumption. (b) Consider maximizing x subject to $2x \leq 1$ and $x \in \mathbb{Z}_{\geq 0}$. Its optimum is zero. The integer version of the corresponding dual minimizes y subject to $2y \geq 1$ and $y \in \mathbb{Z}_{\geq 0}$, whose optimum is one. Both optima are finite and feasible, so strong equality fails. (c) The integer primal feasible set is contained in the real primal set, so $IP \leq P$. The integer dual feasible set is contained in the real dual set and the objective is minimized, so $D \leq ID$. By the real linear-programming strong-duality theorem under the stated feasible, finite optimum assumptions, $P = D$. Combining these gives the claimed chain. In the example it reads $0 \leq 1/2 = 1/2 \leq 1$.

Structural or analytic derivation

Relaxation provides a certified upper bound on an integer maximization, and an integer dual feasible point gives a further upper certificate. Neither algebraic manipulation nor holomorphic continuation restores the missing integrality. The explicit one-variable example isolates the obstruction with no computational experiment.

Comparative cost and preferred method

Weak-duality verification is matrix–vector arithmetic proportional to the nonzero data and gives a certificate without solving the integer program. It does not imply a polynomial-time exact integer optimizer. The native linear and integer optimization methods are retained; there is no justified complex-analytic improvement for the general task.