Source task and conventions

Prove that n distinct relations have $(2n-2)!/(n-1)!$ binary join orders under the source's ordered-tree convention.

Conventional solution and proof

A binary join expression is a rooted ordered full binary tree with n leaves, together with a bijection between its leaves and the n relations. Such a tree has $n-1$ internal nodes: counting edges by parents gives $2I$, while counting a tree's vertices gives $I+n-1$, hence $I = n-1$. The supplied Catalan formula gives $C_{n-1} = \frac{1}{n}\binom{2n-2}{n-1}$ shapes. Every shape admits $n!$ distinct leaf labellings. Their product is

$$n!C_{n-1} = \frac{n!}{n}\frac{(2n-2)!}{((n-1)!)^2} = \boxed{\frac{(2n-2)!}{(n-1)!}}.$$

The tree shape and leaf labelling are recovered uniquely from the expression, proving that neither orders nor labels are overcounted within this convention.

Structural or analytic derivation

Writing J_n for the result, factorial cancellation gives $J_1 = 1$ and $J_n = 2(2n-3)J_{n-1}$. The generating-function Catalan derivation is proved in the binary-tree question and supplies the same count. Commutativity of a relational join does not identify physical left/right plan choices under the source's ordered-plan convention. If one deliberately identifies swapped children, the counting problem changes.

Comparative cost and preferred method