Here is a full minimality proof for the general case. From remainder r, a suffix of length k and value t, $0 \le t < 2^k$, is accepted exactly when $n \mid 2^k r + t$. For $k \ge b$, this condition depends on r only through $r \bmod a$. For $k < b$, divisibility by 2^k forces $t = 0$, after which the condition is $a2^{b-k} \mid r$. Consequently all remainders in the same nonzero residue class modulo a are equivalent. Remainders $r = aj$ split into $b + 1$ classes according to $d = b - \min(b, v_2(j))$, with $d = 0$ for $j = 0$: precisely d trailing zero bits are needed for an immediate accepting zero suffix. There are $(a - 1) + (b + 1) = a + b$ classes. Different residues modulo a are distinguishable: choose $k \ge b$ so large that $2^k \ge n$ and choose the unique $t \in [0, n)$ satisfying $2^k r + t \equiv 0 \pmod{n}$. The same suffix fails for a different residue because a is odd. Among the zero-residue classes, the suffix 0^{d_1} distinguishes classes with $d_1 < d_2$. Every class is reachable by a nonempty binary representation of a representative remainder, allowing the word zero for remainder zero. If $b \ge 1$, the nonaccepting initial residual is equivalent to remainder $n/2$: after the first bit both have the same remainders as a zero-valued prefix, and both reject the empty suffix. No extra state is needed. If $b = 0$, the initial residual is distinct from accepting remainder zero and from every nonzero remainder: a sufficiently long all-zero suffix is accepted from the start but not from any nonzero remainder, since n is odd. Thus

$$\boxed{\text{minimum states} = \begin{cases} a + 1, & b = 0, \\ a + b, & b \ge 1. \end{cases}}$$

This includes $n = 1$: two states recognize all nonempty binary words.

Comparative cost and preferred method

A literal remainder DFA with a separate start has $n + 1$ states. The proved quotient uses $a + b$ states for even n, a ratio $(n + 1)/(a + b)$ in a complete two-symbol transition table, and $n + 1$ for odd n. Recognition remains linear in input length, while storage is reduced. This is an algebraic finite-state optimum certified by distinguishable residual languages; complex analysis is unnecessary.