Conventional solution and proof

Since $L(\phi) = \varnothing$, set union gives $L(\phi \mid s) = \varnothing \cup L(s) = L(s)$. Concatenating with the empty set produces no pairs of words, so $L(\phi s) = L(s\phi) = \varnothing$. The star is the union of all finite concatenation powers, including the zeroth power $\{\varepsilon\}$; every positive power of the empty language is empty. Hence $L(\phi^*) = \{\varepsilon\}$. For a fragment representing ϕ, create an entry and an exit with no path between them. It accepts no string, and can be inserted into the same union, concatenation and star constructions. The complete NFA for the empty language can be a single nonaccepting start state with no transitions.

Structural or analytic derivation

The extension admits dead states: states from which no accepting state is reachable. In minimization of a partial DFA, a missing transition and a transition to a dead state represent the same rejecting continuation, so treating them as different can prevent legitimate mergers. Either remove dead states before partial-DFA minimization, or complete every missing transition to one rejecting sink and apply the usual total-DFA equivalence refinement. All dead states then have identical empty right language and merge.

Comparative cost and preferred method

Coaccessibility can be found by reverse graph search from accepting states in $O(|Q|+|E|)$ time. Removing dead states or adding one sink has linear or constant-state overhead. This preprocessing restores the assumptions of the conventional minimizer. No analytic substitute is more efficient for the source's language-semantic requirement.