Source task and conventions

Determine every admissible value of p_A.

Conventional solution and proof

For $0 < p_A < 1$, all three states communicate: $A \to B$, $B \to C$, $C \to A$ have positive probability. Returns to A of length two (A, B, A) and three (A, B, C, A) both have positive probability. Their greatest common divisor is one, so the irreducible finite chain is aperiodic and hence ergodic. At $p_A = 0$, the chain is the deterministic three-cycle $A \to B \to C \to A$ and has period three. At $p_A = 1$, C is transient and the closed A, B class alternates with period two; the full chain is also reducible. Thus, under the source's irreducible-and-aperiodic meaning,

$$\boxed{0 < p_A < 1.}$$

The stationary distribution for an interior parameter is $\pi_A = \pi_B = 1/(3 - p_A)$ and $\pi_C = (1 - p_A)/(3 - p_A)$, obtained directly from the balance equations.

Structural or analytic derivation

Cycle lengths identify aperiodicity without computing eigenvalues. The stationary formula supplies a separate normalization and balance check but cannot replace the period argument at the endpoints.

Comparative cost and preferred method

A finite graph-reachability and cycle-gcd test proves the classification exactly. Numerical simulation near $p_A = 0$ or 1 can mix slowly and would not establish the endpoint distinction. The standard graph method is already the appropriate efficient method.