Source task and conventions

Prove the claim for the source's uniform teleport operation and state the essential assumptions.

Conventional solution and proof

Let W be a row-stochastic link matrix, with a probability row assigned to every dangling vertex. For $0 < \alpha \leq 1$, uniform teleportation gives $P = (1-\alpha)W + \alpha \mathbf{1}\mathbf{1}^\top/N$, so $P_{ij} \geq \alpha/N > 0$. Every state reaches every other in one step and every state has a self-loop. Therefore the finite chain is irreducible and aperiodic. This proves ergodicity for every directed link graph, including a graph with no edges after dangling rows are completed. A separate quantitative proof uses distributions u, v:

$$\|P^\top u - P^\top v\|_1 = (1-\alpha)\|W^\top (u-v)\|_1 \leq (1-\alpha)\|u-v\|_1.$$