Source task and conventions

Prove the pagewise bound and quantify the $\alpha \to 1$ limit.

Conventional solution and proof

Stationarity gives $\pi = (1-\alpha)W^\top \pi + \alpha u$, $u = \mathbf{1}/N$. Each component of the first term is nonnegative, so $\pi_i \geq \alpha/N$. Since the other $N-1$ components have the same lower bound,

$$\pi_i \leq 1 - (N-1)\alpha/N = \alpha/N + 1 - \alpha.$$

Hence $|\pi_i - \pi_j| \leq 1-\alpha$ for every pair. More precisely, $\pi = \alpha u + (1-\alpha)w$ for the probability vector $w = W^\top \pi$, yielding

$$\|\pi - u\|_\infty \leq (1-\alpha)(1-1/N), \qquad \|\pi - u\|_1 \leq 2(1-\alpha)(1-1/N).$$

Thus all scores tend to $1/N$ and pairwise contrasts vanish at least linearly. The elementary probability-simplex bounds prove all three inequalities.

Structural or analytic derivation

Teleportation is an explicit convex mixture. This representation yields parameter-uniform bounds without computing the particular graph's stationary vector.

Comparative cost and preferred method

The score bounds require $O(1)$ scalar work once N, α are given, whereas solving all N scores requires graph access. The simpler output is a bound, not the complete ranking. Near-uniform scores do not make their exact relative ordering automatically irrelevant or numerically certified.