Source task and conventions

Construct and identify the stationary chain for a mixture of two topic interests.

Conventional solution and proof

Let W be the common row-stochastic link matrix, v_s, v_p the respective teleport probability vectors, $0 < \alpha \leq 1$, and $0 \leq q \leq 1$. The described experiment first chooses whether to teleport with probability α; conditional on teleporting it chooses sports with probability q and politics with probability $1 - q$, then a page according to the corresponding topic vector. Therefore its transition matrix is

$$P_q = (1 - \alpha)W + \alpha \mathbf{1}[qv_s + (1 - q)v_p]^\top.$$

Let $v_q = qv_s + (1 - q)v_p$. The affine map $T_q x = (1 - \alpha)W^\top x + \alpha v_q$ preserves probability vectors and is a contraction with factor $1 - \alpha$, so it has exactly one stationary probability vector. The proposed personalized vector $q\pi_s + (1 - q)\pi_p$ satisfies

$$T_q(q\pi_s + (1 - q)\pi_p) = qT_s\pi_s + (1 - q)T_p\pi_p = q\pi_s + (1 - q)\pi_p.$$

It is thus the steady-state distribution of precisely the random walk in the question. The endpoints $q = 0, 1$ reduce to the corresponding single topic.

Structural or analytic derivation

The same result follows from linearity of $\alpha[I - (1 - \alpha)W^\top]^{-1}v$. A common link and dangling-node policy is essential. If the link operator itself changes with personalization, this displayed affine identity needs to be rederived for that model.