Source task and conventions

Prove the correct ergodicity statement and resolve the full-web qualification.

Conventional solution and proof

Let $S_q = \operatorname{supp}(qv_s + (1-q)v_p)$ and Y_q its forward-reachable closure in the completed link graph for $0 < \alpha < 1$. Every state in Y_q can teleport to any seed in S_q, and from a suitable seed a link walk reaches any other state of Y_q. Thus the restricted chain on Y_q is irreducible. A seed has a positive teleport self-loop, so this irreducible class is aperiodic. It is closed under links and teleportation. Every state outside it has probability at least α per step of entering S_q; the probability of avoiding it for t steps is at most $(1-\alpha)^t$. Hence there is one recurrent ergodic class and one globally attracting stationary distribution, zero outside Y_q.

The literal assertion that the entire web chain is irreducible is false without reachability of every page. For example, take three self-linked pages, sports seed 1 and politics seed 2, with $0 < q < 1$. Page 3 cannot be reached from either seed, so the full chain is reducible. At $\alpha = 1$, the stationary vector is the teleport vector and its positive-support class is again ergodic. Thus the personalized stationary distribution exists and is unique under the source model, but full-state irreducibility requires Y_q to equal the entire web.

Structural or analytic derivation

Contraction proves convergence and uniqueness on the full probability simplex even when states outside the recurrent class are transient. Separating this fact from irreducibility avoids an incorrect use of the word ergodic.

Comparative cost and preferred method