Source task and conventions

Prove the equality of every stationary coordinate in Exercise 21.17.

Conventional solution and proof

For each page i, the two stationary equations are

$$\pi_s(i) = (1-\alpha)\sum_j W_{ji}\pi_s(j) + \alpha v_s(i), \quad \pi_p(i) = (1-\alpha)\sum_j W_{ji}\pi_p(j) + \alpha v_p(i).$$

Multiply the first by q, the second by $1-q$, and add. The vector $r_i = q\pi_s(i) + (1-q)\pi_p(i)$ then satisfies the personalized stationary equation. It is a probability vector. Uniqueness follows because if r, r' are two fixed points, $\|r - r'\|_1 \le (1-\alpha)\|r - r'\|_1$, forcing equality to zero for $\alpha > 0$. Therefore the stationary probability is exactly the claimed value at every page, including pages outside one or both topic supports.