Source task and conventions

Compute one iteration using the sequential convention stated beneath Equation (21.8).

Conventional solution and proof

Let A be the directed adjacency matrix and initialize $h^{(0)} = a^{(0)} = \mathbf{1}$. The source states that hub scores are recomputed first and authority scores then use the recomputed hubs. Thus

$$h^{(1)} = A\mathbf{1} = d_{\text{out}}, \qquad a^{(1)} = A^\top h^{(1)} = A^\top A\mathbf{1}.$$

For a node v, the hub score is its out-degree and the authority score is $\sum_{u \to v} \deg^+(u)$. With weighted links, replace degrees by weighted row sums and include the incoming weight in the latter sum. If an implementation uses simultaneous updates from the two old vectors instead, the first authority score is the in-degree $A^\top\mathbf{1}$; that is a different convention. Any stated normalization then divides the entire corresponding vector by its chosen norm, without changing within-vector rankings.

Structural or analytic derivation

The two sparse products implement $A^\top A$ without forming a potentially dense co-citation matrix. This preserves the source's sequential semantics and explains the second-order authority count.

Comparative cost and preferred method

The first sequential update takes $O(E + N)$ work and vector storage; explicitly multiplying matrices to form $A^\top A$ can create $O(N^2)$ fill-in. Sparse composition is the appropriate standard implementation and the structural method confirms it.