Conventional solution and proof

The displayed matrix is $C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$. Its third row is the sum of its first two rows, so rank is at most two. Its upper-left 2×2 minor has determinant one, so rank is at least two. Therefore $\operatorname{rank} C = 2$. The source calls this matrix diagonal, but its displayed entries are not diagonal; the displayed matrix is the object solved here.

Structural or analytic derivation

A dependence witness proves the upper bound and one nonzero minor proves the lower bound. These are complementary certificates and do not require approximate singular values.

Comparative cost and preferred method

A few exact row operations establish the answer. For a large numerical matrix a rank threshold requires a noise/tolerance model, but that issue is absent in this integer example. The direct algebraic method is preferable.