Source task and conventions

Verify $\lambda = 2$ for $C = \left(\begin{smallmatrix} 6 & -2 \\ 4 & 0 \end{smallmatrix}\right)$ and find its eigenvectors.

Conventional solution and proof

The characteristic determinant is $(6-\lambda)(-\lambda)+8 = \lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4)$, so 2 is an eigenvalue. Solving $(C-2I)v = 0$ gives $4v_1 - 2v_2 = 0$ in both rows. Thus every corresponding eigenvector is $v = t(1,2)^\top$ with $t \neq 0$. Direct multiplication gives $C(1,2)^\top = (2,4)^\top = 2(1,2)^\top$. A unit eigenvector is $(1,2)^\top/\sqrt{5}$.

Structural or analytic derivation

The residual $Cv - 2v = 0$ is an exact certificate for a proposed nonzero eigenvector; the nullspace calculation identifies the entire eigenspace.

Comparative cost and preferred method

For this 2×2 problem direct algebra is minimal. A general eigensolver provides no relevant asymptotic advantage and introduces unnecessary rounding.